|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  | | --- | --- | | **1.** | Find the indicated sum. Show your work.  Solution  **Answer**: 2. | | **2.** | Locate the foci of the ellipse. Show your work.  Solution  The center is (0,0).  The *x-*axis is the major axis. We know this because theterm has the larger denominator.  We find the foci using the following equation and solving for *c*.  The coordinates for the foci are :  **Answer**: | | **3.** | Solve the system by the substitution method. Show your work.  Solution  Use the first equation to express *x* in terms of *y*:  Substitute this expression for *x* in the second equation:  Expand and simplify  or  Now that you know the value for *y*, find the value for *x*  **Answer**: | | **4.** | Graph the function. Then use your graph to find the indicated limit. You do not have to provide the graph  ,  Solution    Graph  From the graph, it can be seen that when *x* approaches 5 from left, *f(x)* approaches 22 and when a approaches from right, *f(x)* approaches 22. Similarly, . Hence,  **Answer**: 22. | | **5.** | Use Gaussian elimination to find the complete solution to the system of equations, or state that none exists. Show your work.  Solution  First, we transform this system into an equivalent system in which we interchange the first equation and the second equation.   |  |  | | --- | --- | |  | (a) |   Next, we eliminate the variable x from all equations except the first:  Replace the second equation in (a) by the sum of the first equation+the second equation:  Replace the third equation in (a) by the sum of the first equation+the third equation:  Then the system is:   |  |  | | --- | --- | |  | (b) |   Replace the third equation in (b) by the sum of the second equation+the third equation:  Then the system is:   |  |  | | --- | --- | |  | (c) |   The last equation. This is not possible. So the system has no solutions; it is not possible to find values *x*, *y*, and *z* that satisfy all three equations simultaneously.  **Answer**: no solutions. | | **6.** | Solve the system of equations using matrices. Use Gaussian elimination with back-substitution.  Solution  The system may be represented by the matrix  If we now reverse the conversion process and turn the augmented matrix into a system of equations we have  Eliminating *z* from all equations except the third. Replace the first equation by the sum of the third equation+the first equation. Replace the second equation by the sum of the third equation+the second equation.  Replace the first equation by the sum of the second equation+the first equation.  Then leads to the system  **Answer:** (1,-4,-2).  **7**. A woman works out by running and swimming. When she runs, she burns  7 calories per minute. When she swims, she burns 8 calories per minute. She wants to burn at least 336 calories in her workout. Write an inequality that describes the situation. Let x represent the number of minutes running and y the number of minutes swimming. Because x and y must be positive, limit the borders to quadrant I only.  Solution  As *x* represent the number of minutes running and when a woman runs, she burns 7 calories per minute then the expression for the number of calories she burns when she runs is .  As *y* represent the number of minutes swimming and when a woman swims, she burns 8 calories per minute then the expression for the number of calories she burns when she swims is .  Therefore expression for the total number of calories she burns is  We want this quantity to be at least 336, so our inequality is  Expressing *y* in terms of *x* we have  Because x and y must be positive, limit the borders to quadrant I only, i.e.  On the plot below we can see the solution that describes the situation.    **Answer:** , . |   **8**. A statement Sn about the positive integers is given. Write statements S1, S2, and S3, and show that each of these statements is true. **Show your work.**  Solution  Using the left part of this statement, find S1, S2, and S3  Using the formula in the right part of the equality, show that each of these statements is true  Hence  Thus we showed that the left side corresponds to the right side in each case, i.e. S1, S2, S3 are true.  Then we can write all expression for each of these statements:    **Answer**:  **9.** A statement Sn about the positive integers is given. Write statements Sk and Sk+1, simplifying Sk+1 completely. Show your work.  -  Solution  According to the question  Find for this we replace all occurrences of *n* by *k* in . That is,  Find for this we replace all occurrences of *k* by *k+1* in . That is,  Simplifying Sk+1 completely we have  **Answer:** ,  .  **10**. Joely's Tea Shop, a store that specializes in tea blends, has available 45 pounds of A grade tea and 70 pounds of B grade tea. These will be blended into 1 pound packages as follows: A breakfast blend that contains one third of a pound of A grade tea and two thirds of a pound of B grade tea and an afternoon tea that contains one half pound of A grade tea and one half pound of B grade tea. If Joely makes a profit of $1.50 on each pound of the breakfast blend and $2.00 profit on each pound of the afternoon blend, how many pounds of each blend should she make to maximize profits? What is the maximum profit?  Solution  The question ask for the number of pounds of each blend, so our variables will stand for that:  *x*: number of pounds blended in the morning  *y*: number of pounds blended in the afternoon  Naturallyand .  According to the question: “A breakfast blend contains one third of a pound of A grade tea and an afternoon tea contains one half pound of A grade tea”. And “Joely's Tea Shop has available 45 pounds of A grade tea”.  And also: “A breakfast blend contains two thirds of a pound of B grade tea and an afternoon tea contains one half pound of B grade tea”. And “Joely's Tea Shop has available 70 pounds of B grade tea”.  Thus we have the inequalities:  The profit relation will be our optimization equation:  The feasibility region graphs as:    When we test the corner points at A (0, 90), B (75, 40), C (105, 0), we obtain the maximum value of *P* = 192.5 at (*x*, *y*) = (75, 40). That is, the solution is "75 pounds blended in the morning and 40 pounds blended in the afternoon".  **Answer:** She should make75 pounds in the morning and 40 pounds in the afternoon to maximize profits. The maximum profit is $192.5.  **11.** Your computer supply store sells two types of laser printers. The first type, A, has a cost of $86 and you make a $45 profit on each one. The second type, B, has a cost of $130 and you make a $35 profit on each one. You expect to sell at least 100 laser printers this month and you need to make at least $3850 profit on them. How many of what type of printer should you order if you want to minimize your cost?  Solution  The question ask for the number of printers of each type, so our variables will stand for that:  *x*: number of the first type’s printers  *y*: number of the second type’s printers  Naturallyand .  According to the question: “You expect to sell at least 100 laser printers this month”, then we have the inequality  And also: “You make a $45 profit on each printer of the first type and a $35 profit on each printer of the second type. You need to make at least $3850 profit on them”.  Thus we have the inequality:  The cost relation will be our optimization equation:  The feasibility region graphs as:    When we test the corner points at A (0, 110), B (35, 65), C (100, 0), we obtain the minimum value of *C* = 8600 at (*x*, *y*) = (100, 0). That is, the solution is "100 printers of the first type".  **Answer:** Ishould order 100 printers of the first type.  **12.** A statement *S*n about the positive integers is given. Write statements S1, S2, and S3, and show that each of these statements is true. **Show your work.**  Solution  Using the left part of this statement, find S1, S2, and S3  Using the formula in the right part of the equality, show that each of these statements is true  Hence  Thus we showed that the left side corresponds to the right side in each case, i.e. S1, S2, S3 are true.  Then we can write all expression for each of these statements:  ; ;  **Answer**: ; ;  .  **13.** Use mathematical induction to prove that the statement is true for every positive integer n. Show your work.  2 is a factor of  Solution  **Proof**. For each , let be the sentence  2 is a factor of  **Step 1**. [We must prove .]  2 is a factor of  so is true.  **Step 2**. Let and assume that is true, i.e., assume that  2 is a factor of (  We want to prove , i.e., that  2 is a factor of  Thus  2 is a factor of (  2 is a factor of  So Thus, by the Principle of Mathematical Induction  For each , 2 is a factor of  **14.** A statement *S*n about the positive integers is given. Write statements S1, S2, and S3, and show that each of these statements is true. **Show your work.**  Sn:   2 is a factor of  Solution  If *n*=1 then S1: 2 is a factor of. It is true because .  If *n*=2 then S2: 2 is a factor of. It is true because .  If *n*=3 then S3: 2 is a factor of. It is true because .  **Answer:** S1: 2 is a factor of ; S2: 2 is a factor of ;  S3: 2 is a factor of .  **15.**     1. What can you conclude about ? How is this shown by the graph? 2. What aspect of costs of renting a car causes the graph to jump vertically by the same amount at its discontinuities?   Solution  According to the graph:  (i.)  (ii.)  (iii.) The limit is not defined because and both exist but . This can be deduced by looking at the graph because it has the discontinuity in the point x=60.  (iv) I think that the jumps and discontinuities in the graph mean that cost of renting a car is is constant only on certain intervals of values x, for example, if the renting of a car is for any amount between 21 and 40, it will cost 44 dollars. If the renting of a car is for any amount between 41 and 60, it will cost 56 dollars, and so on.    **16.** Use mathematical induction to prove that the statement is true for every positive integer *n*.  Solution  **Proof**. For each , let be the sentence  **Step 1**. [We must prove .]  The left part of and the right part of  so is true.  **Step 2**. Let and assume that is true, i.e., assume that  (  We want to prove , i.e., that  The left part of  by  the right part of  So Thus, by the Principle of Mathematical Induction  For each ,  **17.** The following piecewise function gives the tax owed, *T*(*x*), by a single taxpayer on a taxable income of x dollars   |  |  | | --- | --- | | *T*(*x*) = |  |   (i) Determine whether T is continuous at  6061.  (ii) Determine whether T is continuous at 32,473.  (iii) If T had discontinuities, use one of these discontinuities to describe a situation where it might be advantageous to earn less money in taxable income.  Solution  We need to test all the pieces of *T*(*x*) where the indicated points meet.   1. Determine whether T is continuous at  6061.   **is continous at *x*=6061.**   1. Determine whether T is continuous at 32,473.   **is continous at *x*=32,473.**   1. If T had discontinuities, use one of these discontinuities to describe a situation where it might be advantageous to earn less money in taxable income.   This function is continous in all points. But, for example, let it has discontinuity in point x=32,473 ,i.e. ,for example, and then it might be advantageous to earn $32,473 than .  **18.** A statement Sn about the positive integers is given. Write statements Sk and Sk+1, simplifying Sk+1 completely.  Solution  According to the question  Find for this we replace all occurrences of *n* by *k* in . That is,  Find for this we replace all occurrences of *k* by *k+1* in . That is,  Simplifying Sk+1 completely we have  **Answer:**  **19.** An artist is creating a mosaic that cannot be larger than the space allotted which is 4 feet tall and 6 feet wide. The mosaic must be at least 3 feet tall and 5 feet wide. The tiles in the mosaic have words written on them and the artist wants the words to all be horizontal in the final mosaic. The word tiles come in two sizes: The smaller tiles are 4 inches tall and 4 inches wide, while the large tiles are 6 inches tall and 12 inches wide. If the small tiles cost $3.50 each and the larger tiles cost $4.50 each, how many of each should be used to minimize the cost? What is the minimum cost?  Solution  The question ask for the number of tiles of each type, so our variables will stand for that:  *x*: number of the smaller tiles  *y*: number of the large tiles  Naturallyand ,*x* and *y* are integers.  According to the question: “The smaller tiles are 4 inches tall and 4 inches wide, while the large tiles are 6 inches tall and 12 inches wide. The space allotted which is 4 feet tall and 6 feet wide.”  Using a determination of the area of a rectangle find the areas of each type tiles and the space of the mosaic:  the area of the smaller tile is inches,  the area of the large tile is inches,  the area of the space is 3456 inches.  As mosaic that cannot be larger than the space then we can receive the inequality  or  And also: “The mosaic must be at least 3 feet tall and 5 feet wide”, i.e at least 2160 inches.  Thus we have the inequalities:  or 2  The cost relation will be our optimization equation:  The feasibility region graphs as:    When we test the corner points at A (0, 30), B (0, 48), C (216, 0), D(135,0) we obtain the minimum value of *C*= $135 at (*x*, *y*) = (0, 30).  According to the question:”The artist wants the words to all be horizontal in the final mosaic”. Test this. We got the solution: "30 large tiles". The mosaic must be between 3 and 4 feet tall, i.e. between 36 and 48 inches tall and between 5 and 6 feet wide, i.e. between 60 and 72 inches wide. The large tiles are 6 inches tall and 12 inches wide. It is possible if to put down on 6 tiles vertically and on 5 horizontally.  That is, the solution is "30 large tiles".  **Answer**: 30 large tiles should be used to minimize the cost.  **20.** The Fiedler family has up to $130,000 to invest. They decide that they want to have at least $40,000 invested in stable bonds yielding 5.5% and that no more than $60,000 should be invested in more volatile bonds yielding 11%. How much should they invest in each type of bond to maximize income if the amount in the stable bond should not exceed the amount in the more volatile bond? What is the maximum income?  Solution  The question ask for the quantity of investments of each type of bond, so our variables will stand for that:  *x*: quantity of money that was invested in stable bonds  *y*: quantity of money that was invested in more volatile bonds  Naturallyand .  According to the question: “The Fiedler family has up to $130,000 to invest”.  Thus we have the inequality:  And also: They decide that they want to have at least $40,000 invested in stable bonds and that no more than $60,000 should be invested in more volatile bonds. The amount in the stable bond should not exceed the amount in the more volatile bond”.  Thus we have the inequalities:  The income relation will be our optimization equation:  The feasibility region graphs as:    When we test the corner points at A (40000, 60000), B (60000, 60000), C (40000, 40000), we obtain the maximum value of *I* = $9900 at (*x*, *y*) = (60000, 60000). That is, the solution is "$60000 was invested in stable bonds and $60000 was invested in more volatile bonds".  **Answer:** They should invest $60000 in stable bonds and $60000 in more volatile bonds. $9900 is the maximum income. |